Intermediate Microeconomics

Chapter 19: Technology

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Technology

A technology is a process by which inputs are converted to an output.

E.g. labor, a computer, a projector, electricity, and software are being combined to produce this lecture

Usually, several technologies will produce the same product – a blackboard and chalk can be used instead of a computer and a projector.

Which technology is “best”?

How do we compare technologies?

Input Bundles and Outputs

x_i denotes the amount used of input i; i.e. the level of input i.

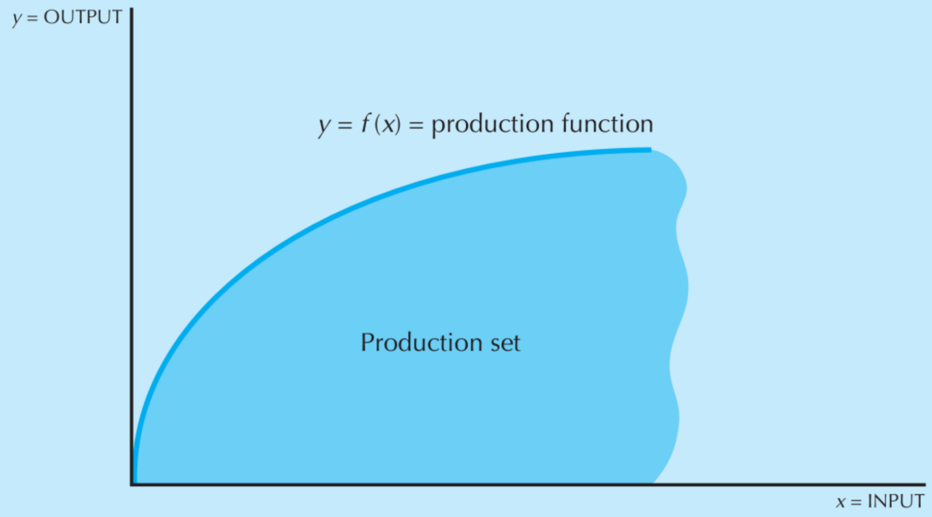
An input bundle is a vector of the input levels; $(\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_n)$.

y denotes the output level.

Production Functions

The technology's production function states the maximum amount of output possible from an input bundle.

$$y = f(x_1, x_2, \ldots, x_n)$$



A production set. Here is a possible shape for a production set.

Technology Sets

A production plan is an input bundle and an output level; $(\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_n, \mathrm{y})$.

A production plan is feasible if

$$\mathrm{y} \leq f(\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_n)$$

The collection of all feasible production plans is the production set (生产集) or technology set (技术集).

Technologies with Multiple Inputs

What does a technology look like when there is more than one input?

The two input case: Input levels are x_1 and x_2. Output level is y.

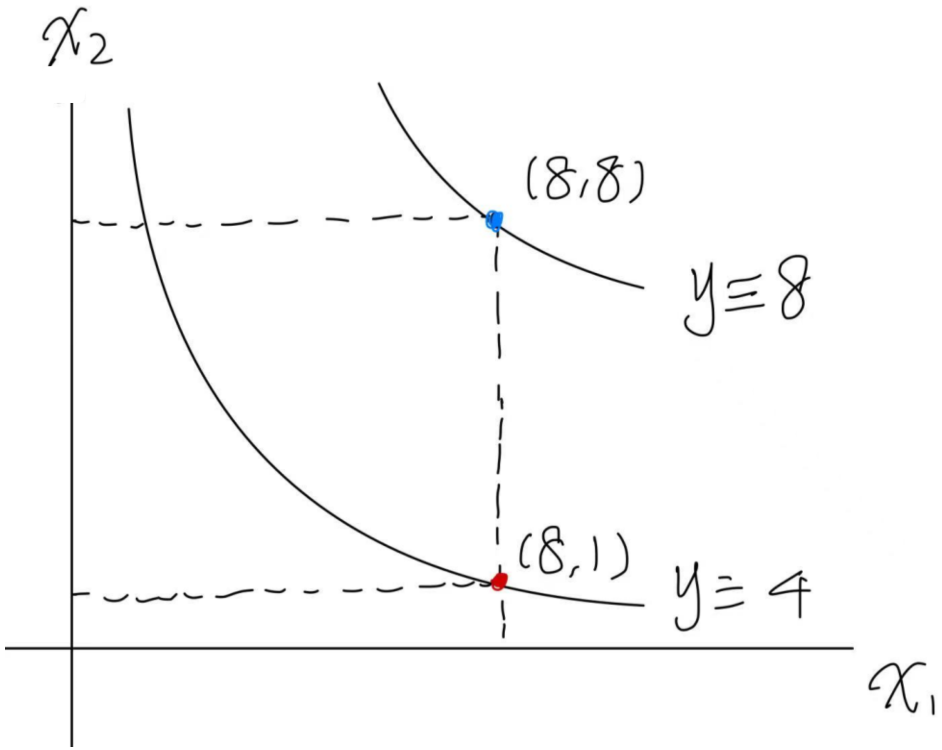
Suppose the production function is

$$y = f(x_1, x_2) = 2x_1^{1/3}x_2^{2/3}$$

E.g. the maximal output level possible from the input bundle
$(x_1, x_2) = (1, 8)$ is $y = 2x_1^{1/3}x_2^{1/3} = 2 \times 1^{1/3} \times 8^{1/3} = 2 \times 1 \times 2 = 4$.
$(x_1, x_2) = (8, 8)$ is $y = 2x_1^{1/3}x_2^{1/3} = 2 \times 8^{1/3} \times 8^{1/3} = 2 \times 2 \times 2 = 8$.

Isoquants with Two Variable Inputs

An isoquant (等产量线) is the set of all possible combinations of inputs 1 and 2 that are just sufficient to produce a given amount of output.



Technologies

1. Cobb-Douglas Technologies:

$$f(\mathrm{x}_1, \mathrm{x}_2) = A\mathrm{x}_1^{\alpha}\mathrm{x}_2^{\beta}$$

2. Fixed-Proportions Technologies:

$$f(\mathrm{x}_1, \mathrm{x}_2) = \min(\alpha\mathrm{x}_1, \beta\mathrm{x}_2)$$

3. Perfect-Substitution Technologies:

$$f(\mathrm{x}_1, \mathrm{x}_2) = \alpha\mathrm{x}_1 + \beta\mathrm{x}_2$$

4. CES Technologies :

$$f(\mathrm{x}_1, \mathrm{x}_2) = [\alpha\mathrm{x}_1^{\rho} + (1-\alpha)\mathrm{x}_2^{\rho}]^{\frac{1}{\rho}}$$

Marginal Products

The marginal product (边际产品) of input i is the rate-of-change of the output level as the level of input i changes, holding all other input levels fixed.

That is,

$$MP_i = \frac{\partial \mathrm{y}}{\partial \mathrm{x}_i}$$

$$MP_1 = \frac{f(\mathrm{x}_1 + \Delta \mathrm{x}_1, \mathrm{x}_2) - f(\mathrm{x}_1, \mathrm{x}_2)}{\Delta \mathrm{x}_1}$$

Returns-to-Scale

规模报酬.

Marginal products describe the change in output level as a single input level changes.

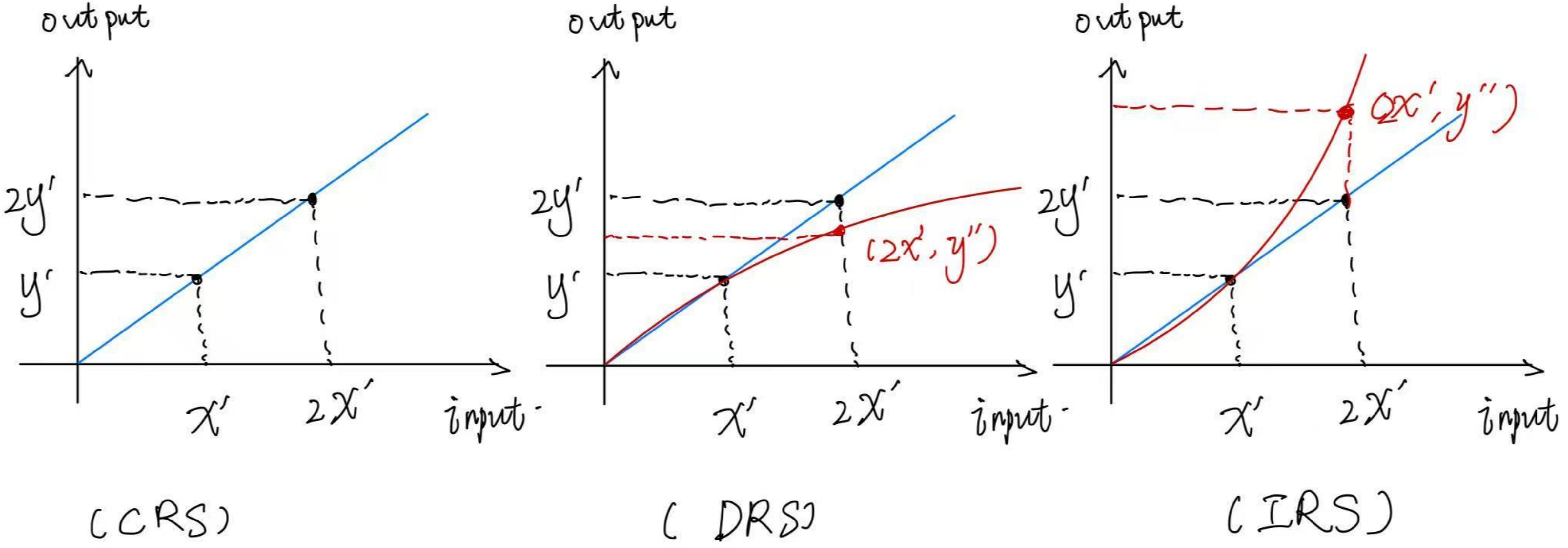
Returns-to-scale describes how the output level changes as all input levels change in direct proportion (e.g. all input levels doubled, or halved).

If, for any input bundle $(x_1, x_2, \ldots, x_n)$,

$$f(kx_1, kx_2, \ldots, kx_n) = kf(x_1, x_2, \ldots, x_n)$$

then the technology described by the production function f exhibits constant returns-to-scale (规模报酬不变, CRS). E.g. (k = 2) doubling all input levels doubles the output level.

Returns-to-Scale



Technical Rate-of-Substitution

if $f(kx_1, kx_2) = k f(x_1, x_2)$.

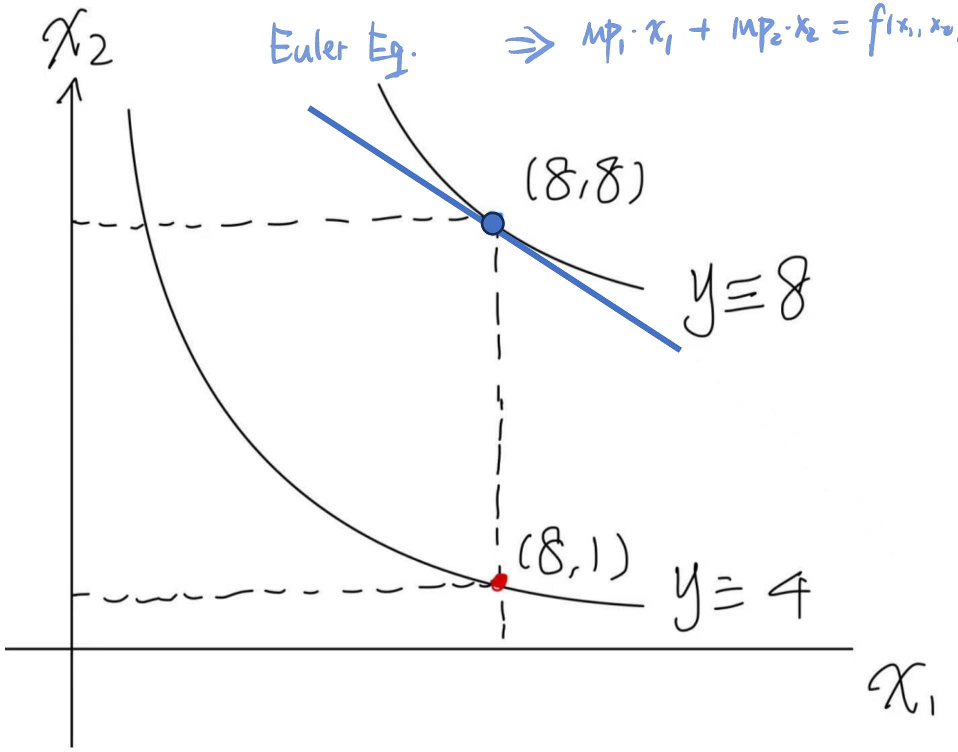
show that $f(x_1, x_2) = MP_1 x_1 + MP_2 x_2$.

take differential to k both sides:

$$\frac{\partial f}{\partial kx_1} \cdot \frac{dkx_1}{dk} + \frac{\partial f}{\partial kx_2} \frac{dkx_2}{dk} = f(x_1, x_2)$$

Euler Eq. $\Rightarrow MP_1 \cdot x_1 + MP_2 \cdot x_2 = f(x_1, x_2)$.

At what rate can a firm substitute one input for another without changing its output level?



The slope is the rate at which input 2 must be given up as input 1's level is increased so as not to change the output level. The slope of an isoquant is its technical rate-of-substitution.

技术替代率.

Technical Rate-of-Substitution

Suppose $y = f(x_1, x_2) = x_1^a x_2^b$

So $\dfrac{\partial y}{\partial x_1} = a x_1^{a-1} x_2^b$ and $\dfrac{\partial y}{\partial x_2} = b x_1^a x_2^{b-1}$

The technical rate-of-substitution is

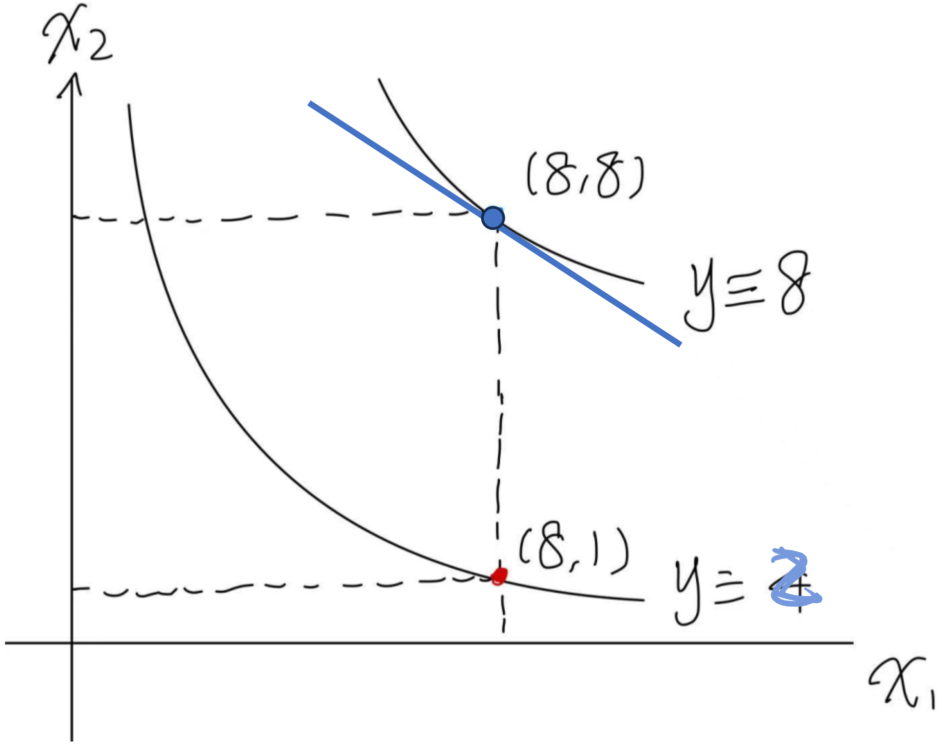
$$\frac{dx_2}{dx_1} = -\frac{\partial y/\partial x_1}{\partial y/\partial x_2} = -\frac{a x_2}{b x_1}$$

An example:

$$y = x_1^{1/3} x_2^{2/3}$$

$$\Rightarrow \text{TRS} = -\frac{a x_2}{b x_1} = -\frac{(1/3) x_2}{(2/3) x_1} = -\frac{x_2}{2x_1}$$

$$\Rightarrow \text{MTRS for } (8,8) = -1/2$$



Elasticity of Substitution

CES: $f(x_1,x_2)=[\alpha x_1^{\rho}+(1-\alpha)x_2^{\rho}]^{\frac{1}{\rho}}$

$\sigma=\frac{d\ln\frac{x_2}{x_1}}{d\ln\left|\frac{dx_2}{dx_1}\right|}=\frac{\left(\frac{1}{x_2}-\frac{1}{x_1}\right)}{\left|\frac{(\rho-1)x_1^{\rho-2}}{\alpha x_1^{\rho-1}}-\frac{(\rho-1)x_2^{\rho-2}}{(1-\alpha)x_2^{\rho-1}}\right|}=\frac{1}{1-\rho}$

$TRS=\frac{\partial y/\partial x_1}{\partial y/\partial x_2}=\frac{\frac{1}{\rho}[\alpha x_1^{\rho}+(1-\alpha)x_2^{\rho}]^{\frac{1}{\rho}-1}\cdot\alpha\rho x_1^{\rho-1}}{\frac{1}{\rho}[\alpha x_1^{\rho}+(1-\alpha)x_2^{\rho}]^{\frac{1}{\rho}-1}\cdot(1-\alpha)\rho x_2^{\rho-1}}=\frac{\alpha x_1^{\rho-1}}{(1-\alpha)x_2^{\rho-1}}$

Elasticity of Substitution measures the ease with which one input can be substituted for another input in the production process, while keeping output constant.

$$\sigma=\frac{d\ln(x_2/x_1)}{d\ln|TRS|}$$

$\ln TRS=\ln\frac{\alpha}{1-\alpha}+(\rho-1)\ln\frac{x_1}{x_2}=\ln\frac{\alpha}{1-\alpha}+(1-\rho)\ln\frac{x_2}{x_1}$

$\frac{d\ln TRS}{d\ln\frac{x_2}{x_1}}=1-\rho\Rightarrow\quad\sigma=\frac{d\ln\frac{x_2}{x_1}}{d\ln TRS}=\frac{1}{1-\rho}.$

If substitution elasticity is

(1) high (like a linear production function , $\sigma\to\infty$), it means that the two inputs are easily interchangeable.

(2) moderate (like a Cobb-Douglas production function, $\sigma=1$), it means that the relative importance of inputs remains constant.

(3) low (like a Leontief production function, $\sigma\to 0$) it means that substituting one input for another is difficult.

Well-behaved Technologies

良态的科技：

单调性、凸性.

A well-behaved technology is monotonic and convex.

Monotonicity: More of any input generates more output.

Convexity: If the input bundles x' and x" both provide y units of output then the mixture ax' + (1-a)x" provides at least y units of output, for any 0 < t < 1.

$f(\alpha x_1 + (1-\alpha)x_2) \geq \alpha f(x_1) + (1-\alpha) f(x_2)$.

Well-behaved Technologies: Convexity